

# Traces of $\Theta^+$ pentaquark in $K^+$ - nucleus dynamics

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Long-standing anomalies in  $K^+$  - nucleus integral cross sections could be resolved by extending the impulse-approximation  $t\rho$  optical-potential framework to incorporate  $K^+$  absorption on pairs of nucleons. Substantially improved fits to the data at  $p_{\text{lab}} \sim 500 - 700$  MeV/c are obtained. An upper bound on the absorption cross section per nucleon is derived,  $\sigma_{\text{abs}}^{(K^+)}/A \sim 3.5$  mb. We conjecture that the underlying microscopic absorption process is  $K^+nN \rightarrow \Theta^+N$ , where  $\Theta^+(1540)$  is the newly discovered exotic  $Y=2, I=0, Z=1$  pentaquark baryon, and estimate that  $\sigma(K^+d \rightarrow \Theta^+p)$  is a fraction of millibarn. Comments are made on the possible strength of the  $\Theta^+$  - nucleus interaction.

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## I. INTRODUCTION AND MOTIVATION

The  $K^+N$  interaction below the pion-production threshold is fairly weak and featureless, as anticipated from an ‘exotic’ channel corresponding to quark content  $qqqq\bar{s}$ , where  $q$  denotes a light nonstrange quark. This merit has motivated past suggestions to probe nuclear in-medium effects by studying scattering and reaction processes with  $K^+$  beams below 800 MeV/c; see Ref.[1] for an early review. Limited total cross-section data [2] on carbon, and elastic and inelastic differential cross section data [3] on carbon and calcium, drew theoretical attention already in the 1980s to the insufficiency of the impulse-approximation  $t\rho$  form of the  $K^+$  - nucleus optical potential, where  $t$  is the free-space  $K^+N$   $t$  matrix, particularly with respect to its reaction content (‘reactivity’ below). In order to account for the increased reactivity in  $K^+$  - nucleus interactions, Siegel *et al.*[4] suggested that nucleons ‘swell’ in the nuclear medium, primarily by increasing the dominant hard-core  $S_{11}$  phase shift. Brown *et al.*[5] suggested that the missing reactivity was due to the reduced in-medium masses of exchanged vector mesons, and this was subsequently worked out in detail in Ref.[6]. Another source for increased reactivity in  $K^+$  - nucleus interactions was discussed in the 1990s and is due to meson exchange-current effects [7, 8].

Some further experimental progress was made during the early 1990s, consisting mostly of measuring attenuation cross sections in  $K^+$  transmission experiments at the BNL-AGS on deuterium and several other nuclear targets in the momentum range  $p_{\text{lab}} = 450 - 740$  MeV/c [9, 10, 11, 12] and of measuring  $K^+$  quasielastic scattering on several targets at 705 MeV/c [13]. New measurements of  $K^+$  elastic and inelastic differential cross sections on C and Ca at 715 MeV/c were reported in Ref.[14] and analyzed in Ref.[15], and self-consistent final values of  $K^+$  integral (reaction and total) cross sections on  ${}^6\text{Li}$ ,  ${}^{12}\text{C}$ ,  ${}^{28}\text{Si}$  and  ${}^{40}\text{Ca}$  were published in Refs.[16, 17]. By the late 1990s, experimentation in  $K^+$  - nuclear physics has subsided, and with it died out also theoretical interest. The subject was reviewed last time in HYP97, concluding that “every experiment of  $K^+$  mesons with complex nu-

clei finds cross sections larger than those predicted” [18]; and based on analyses of  $K^+$  - nuclear integral cross sections [16, 17], it was concluded that “at present theory misses some unconventional in-medium effects” [19].

The  $\Theta^+(1540)$  exotic baryon [20] provides a new mode of reactivity to  $K^+$  - nuclear interactions. The  $\Theta^+$  couples directly to nucleons,  $\Theta^+ \rightarrow K^+n, K^0p$ , but its small width of order 1 MeV or less [21, 22, 23] indicates that this coupling is weak. The  $K^+n$  phase-shift input to the  $t\rho$  optical potential in the vicinity of the  $\Theta^+$  mass (corresponding to  $p_{\text{lab}} \sim 440$  MeV/c) is derived from  $K^+d$  scattering data across the  $\Theta^+$  resonance energy. Therefore, whatever the  $KN\Theta$  coupling is, its effect is already included at least implicitly in the  $t\rho$  impulse approximation. A deuteron target also allows for a two-body production reaction  $K^+d \rightarrow \Theta^+p$  with threshold at  $p_{\text{lab}} \sim 400$  MeV/c [24], but the available  $K^+d$  scattering data do not give any evidence for the opening of such a production channel. It is plausible that a more compact two-nucleon target is required in order to fuse the  $K^+ \bar{s}$  quark together with two  $(ud)_{S=0, I=0}$  diquarks, each of which belongs to a distinct nucleon, into the  $(ud)(ud)\bar{s}$  configuration of the Jaffe-Wilczek model of  $\Theta^+$  [25]. Consequently, nuclear targets denser than the deuteron may offer better chance for  $K^+$  absorption on two nucleons,  $K^+nN \rightarrow \Theta^+N$ , to take place. In the present work we show that traces of such two-body  $K^+$  absorption may be identified in  $K^+$  - nucleus dynamics and, in fact, help resolve the long-standing anomalies in the  $K^+$  - nucleus integral cross sections [16, 17]. We determine the  $K^+$  absorption cross section on nuclei at  $p_{\text{lab}} = 488$  MeV/c which is the closest momentum to the  $\Theta^+$  rest mass, where good data are available. In addition, we discuss briefly the implications of such traces on the  $\Theta^+N$  and  $\Theta^+$  - nucleus interactions in connection with several recent suggestions that the  $\Theta^+$  - nucleus interaction may be sufficiently strong to bind  $\Theta^+$  in nuclei [26, 27]. Our results provide the first concrete demonstration that nuclear targets are potentially useful in the study of exotic baryons.

TABLE I: Fits to the eight  $K^+$ -nuclear integral cross sections at 488 MeV/c.

$V_{\text{opt}}$	$\text{Re}b_0(\text{fm})$	$\text{Im}b_0(\text{fm})$	$\text{Re}B(\text{fm}^4)$	$\text{Im}B(\text{fm}^4)$	$\chi^2/N$
$t\rho$	-0.205(27)	0.173(7)			18.2
	[-0.178]	[0.153]			
Eq.(2)	-0.281(52)	0.133(8)	2.53(37)	0.61(14)	0.67
Eq.(3)	-0.170(58)	0.128(6)	0.12(58)	0.65(7)	0.67

## II. METHODOLOGY, RESULTS AND DISCUSSION

In the calculations presented below the Klein Gordon equation is solved, using the simplest possible  $t\rho$  form for the optical potential

$$2\varepsilon_{\text{red}}^{(A)} V_{\text{opt}}(r) = -4\pi F_A b_0 \rho(r) \quad , \quad (1)$$

where  $\varepsilon_{\text{red}}^{(A)}$  is the reduced energy in the cm system,  $F_A$  is a kinematical factor resulting from the transformation of amplitudes between the  $KN$  and the  $K^+$  - nucleus cm systems and  $b_0$  is the (complex) value of the isospin-averaged  $KN$  scattering amplitude in the forward direction. The Coulomb potential due to the charge distribution of the nucleus is included. This form of the potential takes into account  $1/A$  corrections, an important issue when handling as light a nucleus as  ${}^6\text{Li}$ . Using this approach Friedman *et al.*[16] showed that no *effective* value for  $b_0$  could be found that fits satisfactorily the reaction and total cross sections derived from the BNL-AGS transmission measurements at  $p_{\text{lab}} = 488, 531, 656, 714$  MeV/c on  ${}^6\text{Li}$ ,  ${}^{12}\text{C}$ ,  ${}^{28}\text{Si}$ ,  ${}^{40}\text{Ca}$ . This is demonstrated in the upper part of Fig. 1 for the reaction cross sections per nucleon  $\sigma_R/A$  at 488 MeV/c, where the calculated cross sections using the best-fit  $t\rho$  optical potential are compared with the experimental values listed in Ref. [17]. The values of  $\text{Re}b_0$  and  $\text{Im}b_0$  which specify this  $t\rho$  potential are given in the first row of Table I, whereas the values in square brackets in the second row are for the free-space  $KN$  amplitude. It is seen that fitting the data in terms of an *effective*  $b_0$  requires increasing the *free-space* absorptivity  $\text{Im}b_0$  by  $13 \pm 5\%$ . However, the  $\chi^2/N$  of this density-independent fit is very high. Most of the  $\chi^2$  is due to the impossibility to reconcile the  ${}^6\text{Li}$  data (which for the total cross sections are consistent with the  $K^+d$  ‘elementary’ cross sections) with the data on the other, denser nuclei, as is clearly exhibited in Fig. 1 for the best-fit  $t\rho$  dashed line. If  ${}^6\text{Li}$  were removed out of the data base, then it would have become possible to fit reasonably well the rest of the nuclei, but the rise in  $\text{Im}b_0$  would be substantially higher than the  $13 \pm 5\%$  noted above. Such fits, excluding  ${}^6\text{Li}$ , are less successful at the higher energies.

It was further shown *empirically* [16, 17] that this long-standing problem is resolved if the imaginary (reactive) part of  $V_{\text{opt}}(r)$  is enhanced whenever the average nuclear density  $\bar{\rho} = \frac{1}{A} \int \rho^2 d\mathbf{r}$  exceeds a threshold nuclear density  $\rho_{\text{th}} = 0.088 \pm 0.004 \text{ fm}^{-3}$ . The well-determined value

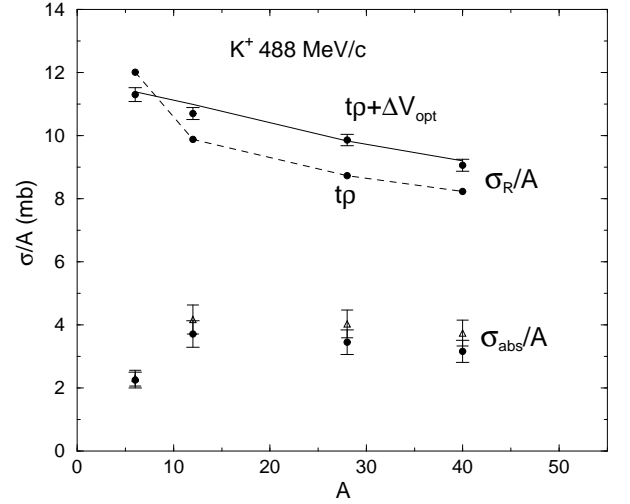


FIG. 1: Data and calculations for  $K^+$  reaction cross sections per nucleon ( $\sigma_R/A$ ) at  $p_{\text{lab}} = 488$  MeV/c are shown in the upper part. Calculated  $K^+$  absorption cross sections per nucleon ( $\sigma_{\text{abs}}/A$ ) are shown in the lower part, see text.

of  $\rho_{\text{th}}$  is considerably higher than the average nuclear density  $\bar{\rho}$  for  ${}^6\text{Li}$ , but is lower than the  $\bar{\rho}$  values appropriate to the other, denser targets. The excellent fit of Refs. [16, 17] clearly suggests that new reactive degrees of freedom open up above the threshold nuclear density of  $0.09 \text{ fm}^{-3}$ . Here we argue that the  $\Theta^+$  provides for such a new degree of freedom via  $K^+$  absorption on two nucleons,  $K^+nN \rightarrow \Theta^+N$ , with threshold at  $p_{\text{lab}} \sim 400$  MeV/c.

We incorporate two-nucleon absorption processes into the impulse-approximation motivated  $V_{\text{opt}}(r)$ , Eq. (1), by adding a  $\rho^2(r)$  piece, as successfully practised in pionic atoms to account for  $\pi^-$  absorption on two nucleons:

$$b_0 \rho(r) \rightarrow b_0 \rho(r) + B \rho^2(r) \quad . \quad (2)$$

Using this potential we have repeated fits to all 32 data points for the reaction and total cross sections. The resulting fit at 488 MeV/c is very good and is shown in Table I. However, the fits at the higher momenta are not as successful, suggesting that one needs an effective way to distinguish between  ${}^6\text{Li}$  and the denser nuclear targets. The average nuclear density  $\bar{\rho}$  provides such discrimination via the ansatz

$$b_0 \rho(r) \rightarrow b_0 \rho(r) + B \bar{\rho} \rho(r) \quad . \quad (3)$$

The resulting fit at 488 MeV/c is also shown in Table I and is equally good to the one using the extension of  $V_{\text{opt}}$  given by Eq. (2). The improvement in the fits to the data with respect to fits using the  $V_{\text{opt}} = t\rho(r)$  form is very substantial, although at momenta higher than 488 MeV/c it is not as spectacular as for the phenomenological approach of Refs. [16, 17]. In Table I we note that  $\text{Im}b_0$  is very well determined and that, in contrast to  $\text{Re}b_0$ , its best-fit values for the two different

TABLE II: A family of fits to  $K^+$ -nuclear integral cross sections at several laboratory momentum  $p_{\text{lab}}$  (in MeV/c), using Eq. (3), with  $\text{Re}b_0$  constrained at its free-space value.

$p_{\text{lab}}$	$\text{Re}b_0(\text{fm})$	$\text{Im}b_0(\text{fm})$	$\text{Re}B(\text{fm}^4)$	$\text{Im}B(\text{fm}^4)$	$\chi^2/N$
488	-0.178	0.126(4)	0.19(11)	0.67(6)	0.69
531	-0.172	0.144(7)	0.50(28)	0.82(9)	6.06
656	-0.165	0.203(5)	2.16(19)	0.78(8)	1.42
714	-0.161	0.218(8)	1.75(41)	0.97(12)	2.40

good fits shown agree with each other within the errors. Thus, the splitting of  $\text{Im}V_{\text{opt}}$  into its two reactive components appears well determined by the data and perhaps is even model independent. This is not the case for  $\text{Re}V_{\text{opt}}$  where its two components are correlated strongly, largely cancelling each other into a resultant poorly determined  $\text{Re}V_{\text{opt}}$ . These features of  $V_{\text{opt}}$  hold true also at the higher momenta, where the best-fit values of  $\text{Re}b_0$  are far more repulsive than those in free space. Holding  $\text{Re}b_0$  fixed at its free-space value, we obtain the results shown in Table II, where the constrained fit at 488 MeV/c is practically identical with the unconstrained fit for Eq. (3) in Table I. The solid line in the upper part of Fig. 1, marked  $t\rho + \Delta V_{\text{opt}}$ , is due to the latter fit and it is graphically clear that it is a very good fit. We note that the values of  $\text{Im}b_0$  in Table II are close to, but somewhat below the corresponding free-space values, and that the two-nucleon absorption coefficient  $\text{Im}B$  rises slowly with energy as appropriate to the increased phase space available to the underlying two-nucleon absorption process  $K^+nN \rightarrow \Theta^+N$ .

The  $K^+$  nuclear absorption cross section  $\sigma_{\text{abs}}^{(K^+)}$  due to the availability of  $\Theta^+$  - nuclear final states is driven by  $\text{Im}(\Delta V_{\text{opt}})$ , which is the added piece of  $\text{Im}V_{\text{opt}}$  due to a nonzero value of  $\text{Im}B$ . One way to approximate it is to use a DWIA-like expression

$$\sigma_{\text{abs}}^{(K^+)} \sim -\frac{2}{\hbar v} \int \text{Im}(\Delta V_{\text{opt}}(r)) |\Psi_{(\Delta V_{\text{opt}}=0)}^{(+)}(\mathbf{r})|^2 d\mathbf{r} , \quad (4)$$

where the distorted waves  $\Psi_{(\Delta V_{\text{opt}}=0)}^{(+)}$  are calculated discarding  $\Delta V_{\text{opt}}$ . Recall that for  $B = 0$ , replacing in the above integral  $\Delta V_{\text{opt}}(r)$  by  $V_{\text{opt}}(r)$  gives the total reaction cross section in the absence of the  $K^+nN \rightarrow \Theta^+N$  channel. However, the precise expression for the total reaction cross section in the presence of this absorption mode into  $\Theta^+$  - nucleus final states requires the use of the fully distorted waves  $\Psi^{(+)}$ , so that a different approximation for the absorption cross section is given by

$$\sigma_{\text{abs}}^{(K^+)} \sim -\frac{2}{\hbar v} \int \text{Im}(\Delta V_{\text{opt}}(r)) |\Psi^{(+)}(\mathbf{r})|^2 d\mathbf{r} . \quad (5)$$

Calculated absorption cross sections *per target nucleon* at  $p_{\text{lab}} = 488$  MeV/c, induced by the  $K^+nN \rightarrow \Theta^+N$  two-body absorption mode, are shown in the lower part of Fig. 1 for the fit using Eq. (3) for  $V_{\text{opt}}$  in Table I. The triangles are for expression (4) and the solid points are

for expression (5). The error bars plotted are due to the uncertainty in the parameter  $\text{Im}B$ . It is seen that these calculated absorption cross sections, for the relatively dense targets of C, Si and Ca, are proportional to the mass number  $A$ , and the cross section per target nucleon due to  $\text{Im}B \neq 0$  is estimated as close to 3.5 mb. This value should be regarded as an upper bound, since other best-fit potentials yield smaller values. The experience gained from studying  $\pi$ -nuclear absorption [28] leads to the conclusion that  $\sigma_{\text{abs}}(K^+NN)$  is smaller than the extrapolation of  $\sigma_{\text{abs}}^{(K^+)}/A$  in Fig. 1 to  $A = 1$ , and since the  $KN$  interaction is weaker than the  $\pi N$  interaction one expects a reduction not larger than a factor of two, so that  $\sigma_{\text{abs}}(K^+NN) \sim 1 - 2$  mb. We note the considerably smaller absorption cross section per nucleon calculated for  ${}^6\text{Li}$  which, considering its low density, suggests a cross section of order fraction of millibarn for  $K^+d \rightarrow \Theta^+p$ , well below the order 1 mb which as Gibbs has argued recently [22] could indicate traces of  $\Theta^+$  in  $K^+d$  total cross sections near  $p_{\text{lab}} \sim 440$  MeV/c. We assume, for the sake of argument, that  $\sigma(K^+d \rightarrow \Theta^+p) \sim 0.5$  mb and that  $K^+d \rightarrow \Theta^+p$  is driven by the two-nucleon process

$$K^+ n \rightarrow \Theta^+ , \quad \Theta^+ N \rightarrow \Theta^+ N . \quad (6)$$

This value for  $\sigma(K^+d \rightarrow \Theta^+p)$  is much smaller than  $\sigma(\pi^+d \rightarrow pp) \sim 12.5$  mb near the (3,3) resonance energy [28], where the primary two-nucleon absorption mechanism is through  $\Delta$  production,

$$\pi^+ p \rightarrow \Delta^{++} , \quad \Delta^{++} n \rightarrow p p , \quad (7)$$

implying even higher cross section for  $\pi^+d \rightarrow \Delta N$ . We then estimate that

$$\frac{\sigma(\pi^+d \rightarrow \Delta N)}{\sigma(K^+d \rightarrow \Theta^+p)} \sim 25 - 50 , \quad (8)$$

which is over an order of magnitude smaller than the ratio of coupling constants squared  $g_{\pi N \Delta}^2/g_{K N \Theta}^2$  assuming  $\Gamma(\Theta^+ \rightarrow KN) \sim 1$  MeV. This means that the  $\Theta^+N$  interaction could be as strong as, or even stronger than the  $\Delta N$  interaction, the latter is known to be sufficiently strong to lead to a sizable  $\Delta$  - nuclear attraction of depth about 30 MeV [29]. Such a strong  $\Theta^+N$  attraction would lead to  $\Theta^+$  - nuclear bound states in hyponuclei [30].

Another crude estimate for the strength of the  $\Theta^+$  - nuclear interaction may be obtained by considering  $\text{Re}(\Delta V_{\text{opt}})$  which is induced by  $\text{Re}B$ . The values of  $\text{Re}B$  for the fits at 488 MeV/c shown in Table I depend strongly on the type of the fit. Taking a representative value of 1 MeV (which is about twice in magnitude the value of  $\text{Im}B$ ), we get moderate attraction at nuclear-matter densities,

$$\text{Re}\Delta V_{\text{opt}}(\rho = 0.17 \text{ fm}^{-3}) \sim -10 \text{ MeV} . \quad (9)$$

We argue that the  $\Theta^+$  - nuclear potential, which is induced by coupling to the  $KN N$  channel here considered, is of a similar size. This may be seen by using Feshbach's

$P, Q$  approach to the construction of optical potentials [31]. A  $\Theta^+$  - nuclear attractive potential of depth about 10 MeV would lead to binding only over a limited portion of the periodic table, consisting mostly of light nuclei, due to the long-range Coulomb repulsion of  $\Theta^+$  by the core protons. We note that our  $K^+nN \rightarrow \Theta^+N$  absorption mechanism is related to the one suggested recently in Ref. [27] as causing a strong  $\Theta^+$  - nuclear attraction, based on  $K\pi$  two-meson cloud contributions to the self energy of  $\Theta^+$  in nuclear matter.

### III. CONCLUSION

In conclusion, we have argued that the anomalous reactivity established systematically in phenomenological analyses of  $K^+$  - nuclear interactions may be assigned naturally to the  $K^+nN \rightarrow \Theta^+N$  absorption channel with threshold at  $p_{\text{lab}} \sim 400$  MeV/c. We have focussed attention in this first report on the reaction and total cross sections extracted [16, 17] from measurements on  ${}^6\text{Li}$ ,  ${}^{12}\text{C}$ ,  ${}^{28}\text{Si}$ ,  ${}^{40}\text{Ca}$  at  $p_{\text{lab}} = 488$  MeV/c which provides the closest proximity to the  $\Theta^+$  threshold. A full report, extending the discussion to higher energies and to differential cross sections, is under preparation. The analysis

of these data is consistent with an upper limit of about 3.5 mb on the  $K^+$  absorption cross section per nucleon, for  $\Theta^+$  production on the denser nuclei of C, Si, Ca, and indicates a sub-millibarn cross section for  $\Theta^+$  production on deuterium. We urge experimenters to look for the  $K^+d \rightarrow \Theta^+p$  two-body production reaction [24] which requires experimental accuracies of 0.1 mb in cross section measurements. We have made arguments for a medium-strong  $\Theta^+$  - nuclear interaction which is attractive, concluding that it might even slightly bind, but obviously this issue is still far from being settled. Undoubtedly, precise  $K^+$  - nuclear scattering and reaction data would be extremely useful to obtain further, more direct evidence for the presence of the  $\Theta^+$  exotic baryon and its effects in the nuclear medium. In particular,  $K^+$  - nuclear data in the range  $p_{\text{lab}} \sim 300 - 500$  MeV/c would be very helpful to study the onset of strange-pentaquark dynamics in the nuclear medium.

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